RGPVNOTES.IN

Program : **B.Tech**

Subject Name: **Turbo Machinery**

Subject Code: **ME-603**

Semester: **6th**

Unit 4 - ROTARY COMPRESSORS, FAN AND BLOWER

COMPRESSORS

We discussed the basic fluid mechanical principles governing the energy transfer in a fluid machine. A brief description of different types of fluid machines using water as the working fluid was also given in Module 1.However, there exist a large number of fluid machines in practice, that use air, steam and gas (the mixture of air and products of burnt fuel) as the working fluids. The density of the fluids changes with a change in pressure as well as in temperature as they pass through the machines. These machines are called 'compressible flow machines' and more popularly 'turbomachines'. Apart from the change in density with pressure, other features of compressible flow, depending upon the regimes, are also observed in course of flow of fluids through turbomachines. Therefore, the basic equation of energy transfer (Euler's equation, as discussed before) along with the equation of state relating the pressure, density and temperature of the working fluid and other necessary equations of compressible flow, are needed to describe the performance of a turbomachine. However, a detailed discussion on all types of turbomachines is beyond the scope of this book. We shall present a very brief description of a few compressible flow machines, namely, compressors, fans and blowers in this module. In practice two kinds of compressors:centrifugal and axial are generally in use.

CENTRIFUGAL COMPRESSORS

A centrifugal compressor is a radial flow rotodynamic fluid machine that uses mostly air as the working fluid and utilizes the mechanical energy imparted to the machine from outside to increase the total internal energy of the fluid mainly in the form of increased static pressure head.

During the second world war most of the gas turbine units used centrifugal compressors. Attention was focused on the simple turbojet units where low power-plant weight was of great importance. Since the war, however, the axial compressors have been developed to the point where it has an appreciably higher isentropic efficiency. Though centrifugal compressors are not that popular today, there is renewed interest in the centrifugal stage, used in conjunction with one or more axial stages, for small turbofan and turboprop aircraft engines.

Classification based on pressure rise

A centrifugal compressor essentially consists of three components.

1. A **stationary casing**
2. A **rotating impeller** as shown in Fig. 4.1 (a) which imparts a high velocity to the air. The impeller may be single or double sided as show in Fig. 4.1 (b) and (c), but the fundamental theory is same for both.
3. A **diffuser** consisting of a number of fixed diverging passages in which the air is decelerated with a consequent rise in static pressure.

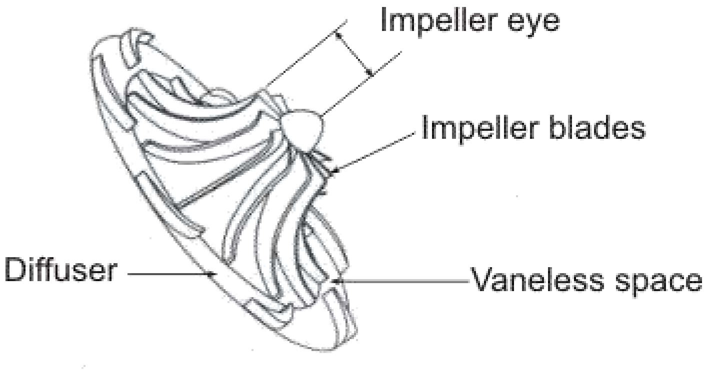
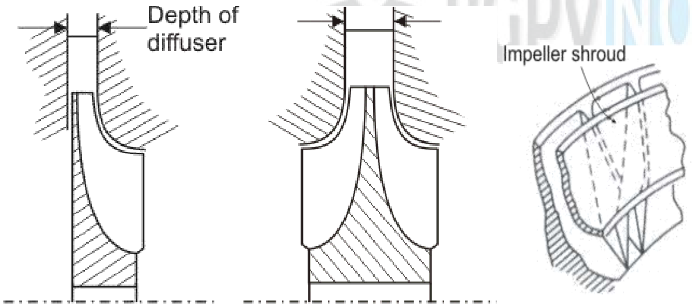


Figure 4.1(a)



(b) (c) (d)

Figure 4.1 Schematic views of a centrifugal compressor

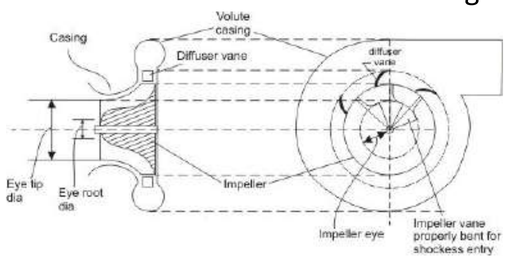


Figure 4.2 Single entry and single outlet centrifugal compressor

Figure 4.2 is the schematic of a centrifugal compressor, where a single entry radial impeller is housed inside a volute casing.

Principle of operation: Air is sucked into the impeller eye and whirled outwards at high speed by the impeller disk. At any point in the flow of air through the impeller the centripetal acceleration is obtained by a pressure head so that the static pressure of the air increases from the eye to the tip of the impeller. The remainder of the static pressure rise is obtained in the diffuser, where the very high velocity of air leaving the impeller tip is reduced to almost the velocity with which the air enters the impeller eye.

Usually, about half of the total pressure rise occurs in the impeller and the other half in the diffuser. Owing to the action of the vanes in carrying the air around with the impeller, there is a slightly higher static pressure on the forward side of the vane than on the trailing face. The air will thus tend to flow around the edge of the vanes in the clearing space between the impeller and the casing. This result in a loss of efficiency and the clearance must be kept as small as possible. Sometimes, a shroud attached to the blades as shown in Figure 4.1(d) may eliminate such a loss, but it is avoided because of increased disc friction loss and of manufacturing difficulties.

The straight and radial blades are usually employed to avoid any undesirable bending stress to be set up in the blades. The choice of radial blades also determines that the total pressure rise is divided equally between impeller and diffuser.

Before further discussions following points are worth mentioning for a centrifugal compressor.

(i) The pressure rise per stage is high and the volume flow rate tends to be low. The pressure rise per stage is generally limited to 4:1 for smooth operations.

(ii) Blade geometry is relatively simple and small foreign material does not affect much on operational characteristics.

(iii) Centrifugal impellers have lower efficiency compared to axial impellers and when used in aircraft engine it increases frontal area and thus drag. Multistaging is also difficult to achieve in case of centrifugal machines.

CENTRIFUGAL AND AXIAL FLOW MACHINES

Work done and pressure rise

Since no work is done on the air in the diffuser, the energy absorbed by the compressor will be determined by the conditions of the air at the inlet and outlet of the impeller. At the first instance, it is assumed that the air enters the impeller eye in the axial direction, so that the initial angular momentum of the air is zero. The axial portion of the vanes must be curved so that the air can pass smoothly into the eye. The angle which the leading edge of a vane makes with the tangential direction α, will be given by the direction of the relative velocity of the air at inlet Vr, as shown in Fig. 4.3. The air leaves the impeller tip with an absolute velocity of V2 that will have a tangential or whirl component Vw .Under ideal conditions Vw, would be such that the whirl component is equal to the impeller speed V1 at the tip. Since air enters the impeller in axial direction, α.

Centrifugal Blowers Vane shape

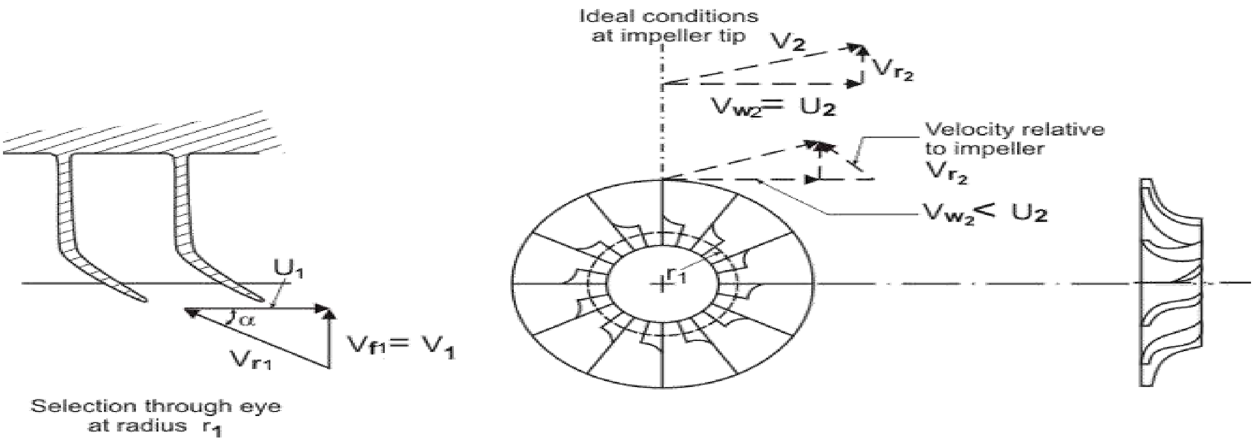


Figure 4.3 Velocity triangles at inlet and outlet of impeller blades

The energy transfer per unit mass of air is given as

$$\frac{E}{m} = U_2^2 \qquad (4.1)$$

Due to its inertia, the air trapped between the impeller vanes is reluctant to move round with the impeller and we have already noted that this results in a higher static pressure on the leading face of a vane than on the trailing face. It also prevents the air acquiring a whirl velocity equal to impeller speed. This effect is known as slip. Because of slip, we obtain Vw2 < U2. The slip factor *σ* is defined in the similar way as done in the case of a centrifugal pump as

$$\sigma = \frac{V_{w2}}{U_2}$$

The value of *σ* lies between *0.9 to 0.92*. The energy transfer per unit mass in case of slip becomes

$$\frac{E}{m} = V_{w2} U_2 = \sigma U_2^2 \quad (4.2)$$

One of the widely used expressions for σ was suggested by Stanitz from the solution of potential flow through impeller passages. It is given by

$$\sigma = 1 - \frac{0.63\pi}{n}$$, where n is the number of vanes.

Power Input Factor

The power input factor takes into account of the effect of disk friction, windage, etc. for which a little more power has to be supplied than required by the theoretical expression. Considering all these losses, the actual work done (or energy input) on the air per unit mass becomes

$$w = \Psi \sigma U_2^2 \qquad (4.3)$$

Where w is the power input factor. From steady flow energy equation and in consideration of air as an ideal gas, one can write for adiabatic work w per unit mass of air flow as

$$w = c_p (T_{0_2} - T_{0_1}) \quad (4.4)$$

where T_{01} and T_{02} are the stagnation temperatures at inlet and outlet of the impeller, and Cp is the mean specific heat over the entire temperature range. With the help of Eq. (4.3), we can write

$$w = \Psi \sigma U_2^2 = c_p (T_{0_2} - T_{0_1}) \quad (4.5)$$

The stagnation temperature represents the total energy held by a fluid. Since no energy is added in the diffuser, the stagnation temperature rise across the impeller must be equal to that across the whole compressor. If the stagnation temperature at the outlet of the diffuser is designated by T03, Then One can write from Eqn. (4.5)

$$\frac{T_{0_2}}{T_{0_1}} = \frac{T_{0_3}}{T_{0_1}} = 1 + \frac{\Psi \sigma U_2^2}{c_p T_{0_1}} \qquad (4.6)$$

The overall stagnation pressure ratio can be written as

$$\frac{p_{03}}{p_{01}} = \left(\frac{T_{03s}}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}}$$

$$= \left[1 + \frac{\eta_c (T_{03} - T_{01})}{T_{01}}\right]^{\frac{\gamma}{\gamma-1}} \qquad (4.7)$$

where, T03 and T03s are the stagnation temperatures at the end of an ideal (isentropic) and actual process of compression respectively (Figure 7.1), and ηc is the isentropic efficiency defined as

$$\eta_c = \frac{T_{03s} - T_{01}}{T_{03} - T_{01}} \qquad (4.8)$$

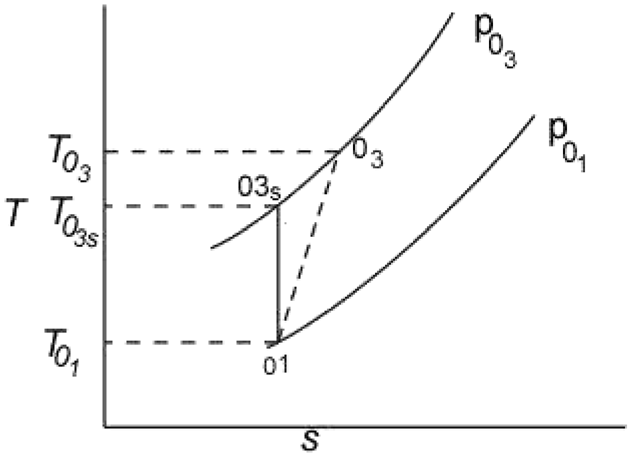


Figure 4.4 Ideal and actual processes of compression on T-s plane

Since the stagnation temperature at the outlet of impeller is same as that at the outlet of the diffuser, one can also write T03s in place of T03 in Eq. (4.8). Typical values of the power input factor lie in the region of 1.035 to 1.04. If we know T03 we will be able to calculate the stagnation pressure rise for a given impeller speed. The variation in stagnation pressure ratio across the impeller with the impeller speed is shown in Figure 4.4. For common material, it is limited to 450 m/s.

Figure 4.5 shows the inducing section of a compressor. The relative velocity Vr at the eye tip has to be held low otherwise the Mach umber given by M will be too high causing shock losses.

Mach number M should be in the range of 0.7-0.9. The typical inlet velocity triangles for large and medium or small eye tip diameter are shown in Figure 4.7(a) and (b) respectively.

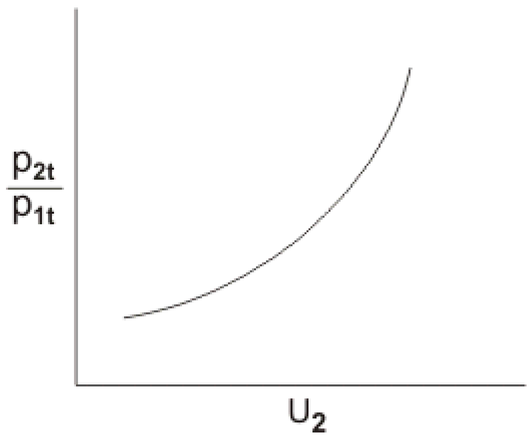


Figure 4.5 Variation in stagnation pressure ratio with impeller tip speed

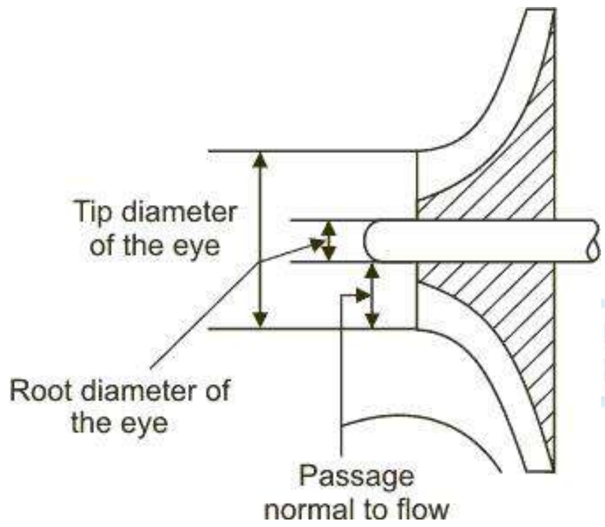


Figure 4.6 Inducing section of a centrifugal compressor

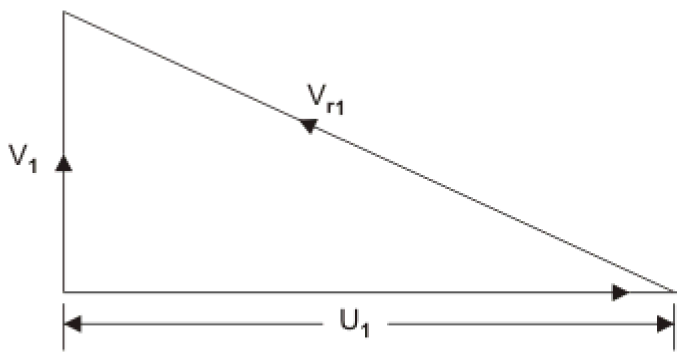


Figure 4.7 (a)

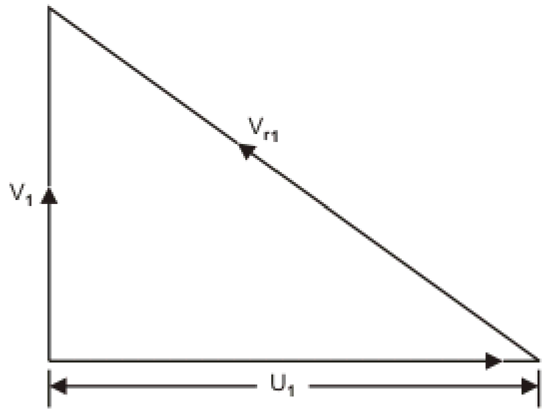


Figure 4.7 (b)

Figure 4.7 Velocity triangles at the tip of eye

DIFFUSER

The basic purpose of a compressor is to deliver air at high pressure required for burning fuel in a combustion chamber so that the burnt products of combustion at high pressure and temperature are used in turbines or propelling nozzles (in case of an aircraft engine) to develop mechanical power. The problem of designing an efficient combustion chamber is eased if velocity of the air entering the combustion chamber is as low as possible. It is necessary, therefore to design the diffuser so that only a small part of the stagnation temperature at the compressor outlet corresponds to kinetic energy.

It is much more difficult to arrange for an efficient deceleration of flow than it is to obtain efficient acceleration. There is a natural tendency in a diffusing process for the air to break away from the walls of the diverging passage and reverse its direction. This is typically due to the phenomenon of boundary layer separation and is shown in Figure. 4.8. Experiments have shown that the maximum permissible included angle of divergence is 11° to avoid considerable losses due to flow separation.

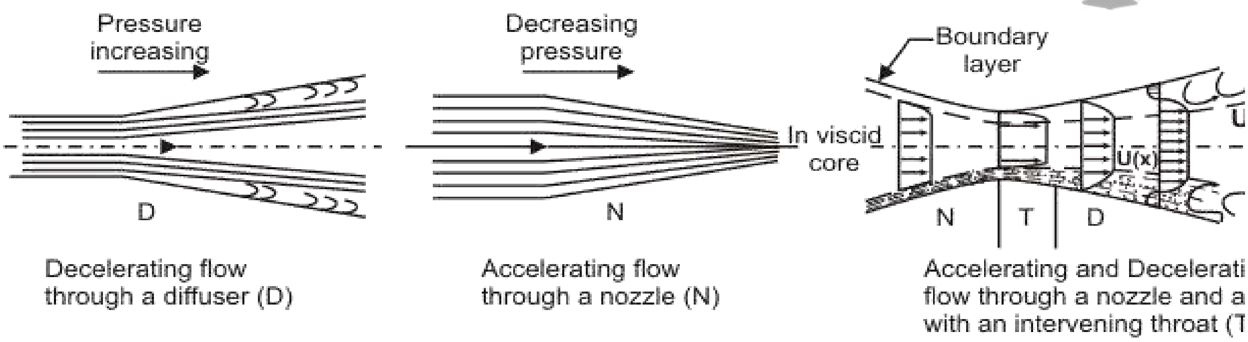


Figure 4.8 Accelerating and decelerating flows

In order to control the flow of air effectively and carry-out the diffusion process in a length as short as possible, the air leaving the impeller is divided into a number of separate streams by fixed diffuser vanes. Usually the passages formed by the vanes are of constant depth, the width diverging in accordance with the shape of the vanes. The angle of the diffuser vanes at the leading edge must be designed to suit the direction of the absolute velocity of the air at the radius of the leading edges, so that the air will flow smoothly over vanes. As there is a radial gap between the impeller tip and the leading edge of the vanes, this direction will not be that with which the air leaves the impeller tip.

To find the correct angle for diffuser vanes, the flow in the vane less space should be considered. No further energy is supplied to the air after it leaves the impeller. If we neglect the frictional losses, the angular momentum ω remains constant. Hence h0 decreases from impeller tip to diffuser vane, in inverse proportion to the radius. For a channel of constant depth, the area of flow in the radial direction is directly proportional to the radius. The radial velocity Vr will therefore also decrease from impeller tip to diffuser vane, in accordance with the equation of continuity. If both h0 and Vr decrease from the impeller tip then the resultant velocity *V* decreases from the impeller tip and some diffusion takes place in the vane less space. The consequent increase in density means that P2 will not decrease in inverse proportion to the radius and the way V2 varies must be found from the equation of continuity.

Losses in a Centrifugal Compressor

The losses in a centrifugal compressor are almost of the same types as those in a centrifugal pump. However, the following features are to be noted.

Frictional losses: A major portion of the losses is due to fluid friction in stationary and rotating blade passages. The flow in impeller and diffuser is decelerating in nature. Therefore the frictional losses are due to both skin friction and boundary layer separation. The losses depend on the friction factor, length of the flow passage and square of the fluid velocity. The variation of frictional losses with mass flow is shown in Figure. 4.9.

Incidence losses: During the off-design conditions, the direction of relative velocity of fluid at inlet does not match with the inlet blade angle and therefore fluid cannot enter the blade passage smoothly by gliding along the blade surface. The loss in energy that takes place because of this is known as incidence loss. This is sometimes referred to as shock losses. However, the word shock in this context should not be confused with the aerodynamic sense of shock which is a sudden discontinuity in fluid properties and flow parameters that arises when a supersonic flow decelerates to a subsonic one.

Clearance and leakage losses: Certain minimum clearances are necessary between the impeller shaft and the casing and between the outlet periphery of the impeller eye and the casing. The leakage of gas through the shaft clearance is minimized by employing glands. The clearance losses depend upon the impeller diameter and the static pressure at the impeller tip. A larger diameter of impeller is necessary for a higher peripheral speed U and it is very difficult in the situation to provide sealing between the casing and the impeller eye tip.

The variations of frictional losses, incidence losses and the total losses with mass flow rate are shown in Figure 4.9

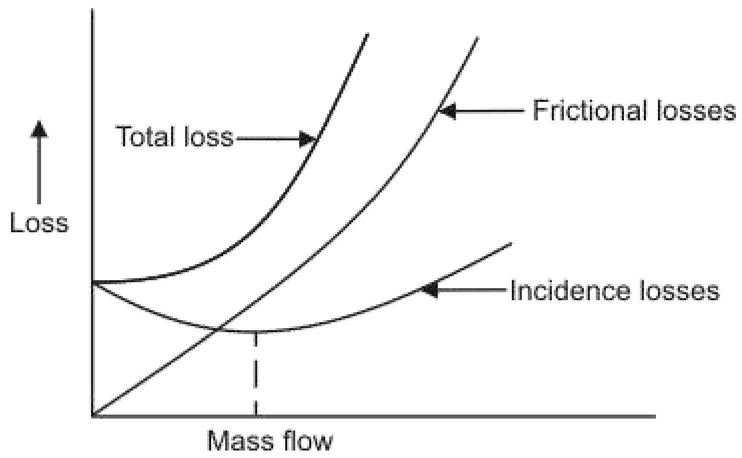


Figure 4.9 Dependence of various losses with mass flow in a centrifugal compressor

The leakage losses comprise a small fraction of the total loss. The incidence losses attain the minimum value at the designed mass flow rate. The shock losses are, in fact zero at the designed flow rate. However, the incidence losses, as shown in Fig. 4.9, comprises both shock losses and impeller entry loss due to a change in the direction of fluid flow from axial to radial direction in the vane-less space before entering the impeller blades. The impeller entry loss is similar to that in a pipe bend and is very small compared to other losses. This is why the incidence losses show a nonzero minimum value (Figure. 4.9) at the designed flow rate.

Compressor characteristics

The theoretical and actual head-discharge relationships of a centrifugal compressor are same as those of a centrifugal pump as described in Module 1. However, the performance of a compressor is usually specified by curves of delivery pressure and temperature against mass flow rate for various fixed values of rotational speed at given values of inlet pressure and temperature. It is always advisable to plot such performance characteristic curves with dimensionless variables. To find these dimensionless variables, we start with a implicit functional relationship of all the variables as

$$F(D, N, m, p_{0_1}, p_{0_2}, RT_{0_1}, RT_{0_2}) = 0 \quad (4.9)$$

Where *D* = characteristic linear dimension of the machine,

N = rotational,

m = mass flow rate,

P01 = stagnation pressure at compressor inlet,

P02 =stagnation pressure at compressor outlet,

T01 = stagnation temperature at compressor inlet,

T02 = stagnation temperature at compressor outlet, and

R = characteristics gas constant.

By making use of Buckingham's π theorem, we obtain the non-dimensional groups (π terms) as

$$\frac{p_{0_2}}{p_{0_1}}, \frac{T_{0_2}}{T_{0_1}}, \frac{m\sqrt{RT_{0_1}}}{D^2 p_{0_1}}, \frac{ND}{\sqrt{RT_{0_1}}}$$

The third and fourth non-dimensional groups are defined as 'non-dimensional mass flow' and 'non-dimensional rotational speed' respectively. The physical interpretation of these two non-dimensional groups can be ascertained as follows.

$$\frac{m\sqrt{RT}}{D^2 p} = \frac{\rho AV\sqrt{RT}}{D^2 p} = \frac{p}{RT}, \frac{AV\sqrt{RT}}{D^2 p} \propto, \frac{V}{\sqrt{RT}} \propto M_F$$

$$\frac{ND}{\sqrt{RT}} = \frac{U}{\sqrt{RT}} \propto M_R$$

Therefore, the 'non-dimensional mass flow' and 'non-dimensional rotational speed' can be regarded as flow Mach number and rotational speed Mach number, .

When we are concerned with the performance of a machine of fixed size compressing a specified gas and *D* may be omitted from the groups and we can write

$$\text{Function} \left(\frac{p_{2t}}{p_{1t}}, \frac{T_{2t}}{T_{1t}}, \frac{m\sqrt{T_{0_1}}}{p_{0_1}}, \frac{N}{\sqrt{T_{0_1}}} \right) = 0 \qquad (4.10)$$

Though the terms P1t and P2t are truly not dimensionless, they are referred as non-dimensional mass flow' and 'non-dimensional rotational speed' for practical purpose. The stagnation pressure and temperature ratios T1t and T2t are plotted against h in the form of two families of curves, each curve of a family being drawn for fixed value. The two families of curves represent the compressor characteristics. From these curves, it is possible to draw the curves of isentropic efficiency ηc. We can recall, in this context, the definition of the isentropic efficiency as

$$\eta_C = \frac{T_{0_{2s}} - T_{0_1}}{T_{0_2} - T_{0_1}} = \frac{(p_{0_2} / p_{0_1})^{\frac{\gamma-1}{\gamma}} - 1}{(T_{0_2} / T_{0_1}) - 1} \qquad (4.11)$$

Before describing a typical set of characteristics, it is desirable to consider what might be expected to occur when a valve placed in the delivery line of the compressor running at a constant speed, is slowly opened. When the valve is shut and the mass flow rate is zero, the pressure ratio will have some value. Figure 4.10 indicates a theoretical characteristics curve ABC for a constant speed.

The centrifugal pressure head produced by the action of the impeller on the air trapped between the vanes is represented by the point 'A' in Figure 4.10. As the valve is opened, flow commences and diffuser begins to influence the pressure rise, for which the pressure ratio increases. At some point '*B*', efficiency approaches its maximum and the pressure ratio also reaches its maximum.

Further increase of mass flow will result in a fall of pressure ratio. For mass flows greatly in excess of that corresponding to the design mass flow, the air angles will be widely different from the vane angles and breakaway of the air will occur. In this hypothetical case, the pressure ratio drops to unity at *'C'*, when the valve is fully open and all the power is absorbed in overcoming internal frictional resistances.

In practice, the operating point 'A' could be obtained if desired but a part of the curve between *'A'* and 'B' could not be obtained due to surging. It may be explained in the following way. If we suppose that the compressor is operating at a point 'D' on the part of characteristics curve (Figure 4.10) having a positive slope, then a decrease in mass flow will be accompanied by a fall in delivery pressure. If the pressure of the air downstream of the compressor does not fall quickly enough, the air will tend to reverse its direction and will flow back in the direction of the resulting pressure gradient. When this occurs, the pressure ratio drops rapidly causing a further drop in mass flow until the point 'A' is reached, where the mass flow is zero. When the pressure downstream of the compressor has reduced sufficiently due to reduced mass flow rate, the positive flow becomes established again and the compressor picks up to repeat the cycle of events which occurs at high frequency.

This surging of air may not happen immediately when the operating point moves to the left of 'B' because the pressure downstream of the compressor may at first fall at a greater rate than the delivery pressure. As the mass flow is reduced further, the flow reversal may occur and the conditions are unstable between 'A' and 'B'. As long as the operating point is on the part of the characteristics having a negative slope, however, decrease in mass flow is accompanied by a rise in delivery pressure and the operation is stable.

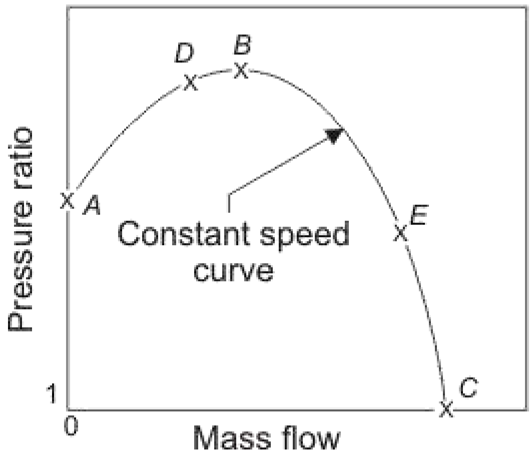


Figure 4.10 The theoretical characteristic curve

There is an additional limitation to the operating range, between 'B' and 'C'. As the mass flow increases and the pressure decreases, the density is reduced and the radial component of velocity must increase. At constant rotational speed this means an increase in resultant velocity and hence an angle of incidence at the diffuser vane leading edge. At some point say 'E', the position is reached where no further increase in mass flow can be obtained no matter how wide open the control valve is. This point represents the maximum delivery obtainable at the particular rotational speed for which the curve is drawn. This indicates that at some point within the compressor sonic conditions have been reached, causing the limiting maximum mass flow rate to be set as in the case of compressible flow through a converging diverging nozzle. Choking is said to have taken place. Other curves may be obtained for different speeds, so that the actual variation of pressure ratio over the complete range of mass flow and rotational speed will be shown by curves such as those in Figure 4.11. The left hand extremities of the constant speed curves may be joined up to form surge line, the right hand extremities indicate choking (Figure 4.11).

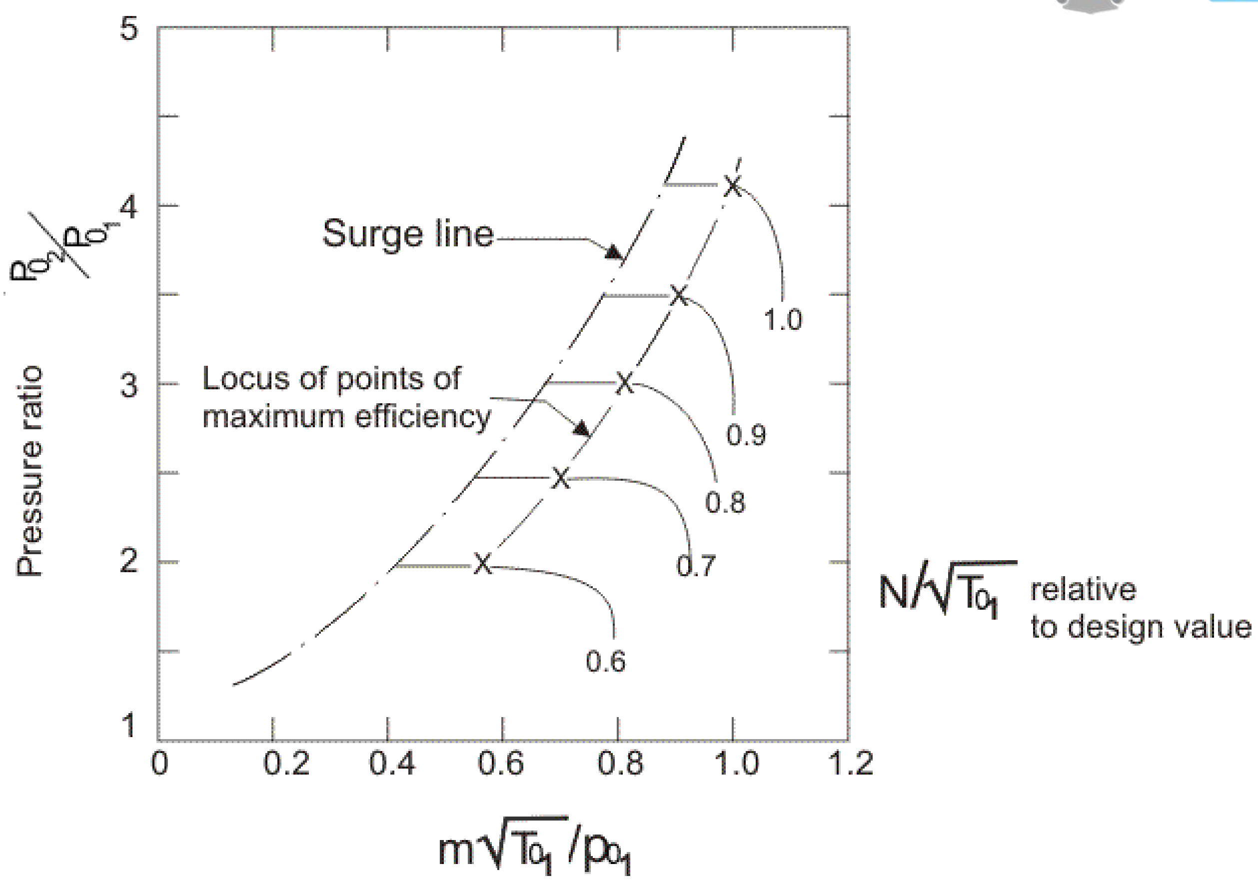


Figure 4.11 Variations of pressure ratio over the complete range of mass flow for different rotational speeds

AXIAL FLOW COMPRESSORS

The basic components of an axial flow compressor are a rotor and stator, the former carrying the moving blades and the latter the stationary rows of blades. The stationary blades convert the kinetic energy of the fluid into pressure energy, and also redirect the flow into an angle suitable for entry to the next row of moving blades. Each stage will consist of one rotor row followed by a stator row, but it is usual to provide a row of so called inlet guide vanes. This is an additional stator row upstream of the first stage in the compressor and serves to direct the axially approaching flow correctly into the first row of rotating blades. For a compressor, a row of rotor blades followed by a row of stator blades is called a stage. Two forms of rotor have been taken up, namely drum type and disk type. A disk type rotor illustrated in Figure 4.11 The disk type is used where consideration of low weight is most important. There is a contraction of the flow annulus from the low to the high pressure end of the compressor. This is necessary to maintain the axial velocity at a reasonably constant level throughout the length of the compressor despite the increase in density of air. Figure 4.12 illustrate flow through compressor stages. In an axial compressor, the flow rate tends to be high and pressure rise per stage is low. It also maintains fairly high efficiency.

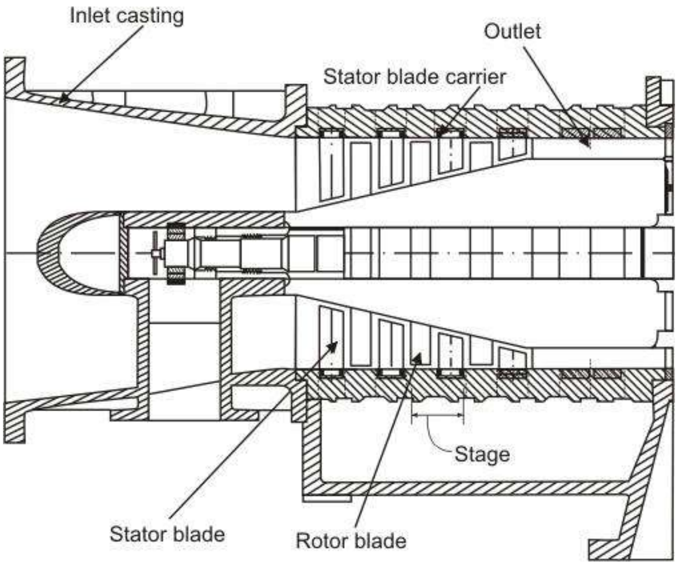


Figure 4.11 Disk type axial flow compressor

The basic principle of acceleration of the working fluid, followed by diffusion to convert acquired kinetic energy into a pressure rise, is applied in the axial compressor. The flow is considered as occurring in a tangential plane at the mean blade height where the blade peripheral velocity is U . This two dimensional approach means that in general the flow velocity will have two components, one axial and one peripheral denoted by subscript w , implying a whirl velocity. It is first assumed that the air approaches the rotor blades with an absolute velocity, V1, and angle α to the axial direction. In combination with the peripheral velocity U of the blades, its relative velocity will be Vr at angle β as shown in the upper velocity triangle (Figure 4.12). After passing through the diverging passages formed between the rotor blades which do work on the air and increase its absolute velocity, the air will emerge with the relative velocity of Vr at angle β which is less than 30 degree. This turning of air towards the axial direction is, as previously mentioned, necessary to provide an increase in the effective flow area and is brought about by the camber of the blades. Since β is less than 30 degree due to diffusion, some pressure rise has been accomplished in the rotor. The velocity V1 in combination with U gives the absolute velocity V2 at the exit from the rotor at an angle α to the axial direction. The air then passes through the passages formed by the stator blades where it is further diffused to velocity Vr2 at an angle β2 which in most designs equals to V2 so that it is prepared for entry to next stage. Here again, the turning of the air towards the axial direction is brought about by the camber of the blades.

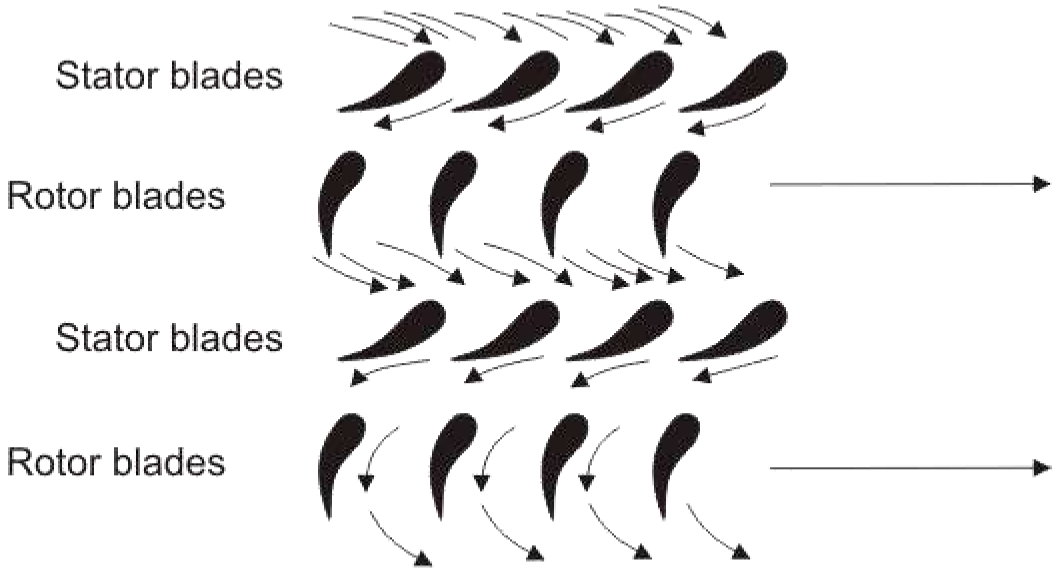


Figure 4.12 Flow through stages

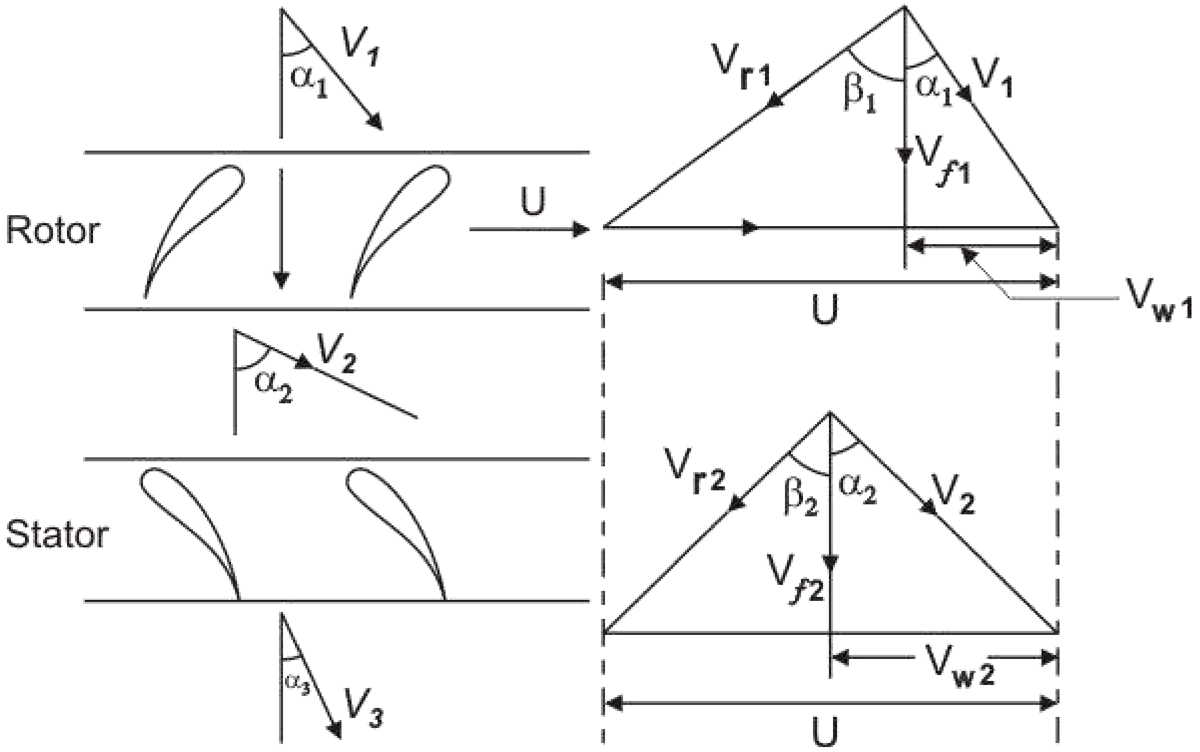


Figure 4.13 Velocity triangles

Two basic equations follow immediately from the geometry of the velocity triangles. These are:

$$\frac{U}{V_f} = \tan\alpha_1 + \tan\beta_1 \tag{4.11}$$

$$\frac{U}{V_f} = \tan\alpha_2 + \tan\beta_2 \tag{4.12}$$

In which Vf is the axial velocity, assumed constant through the stage. The work done per unit mass or specific work input, *w* being given by

$$w = U(V_{w2} - V_{w1}) \quad (4.13)$$

This expression can be put in terms of the axial velocity and air angles to give

$$w = UV_f(\tan\alpha_2 - \tan\alpha_1) \quad (4.14)$$

or by using Eqs. (4.11) and (4.12)

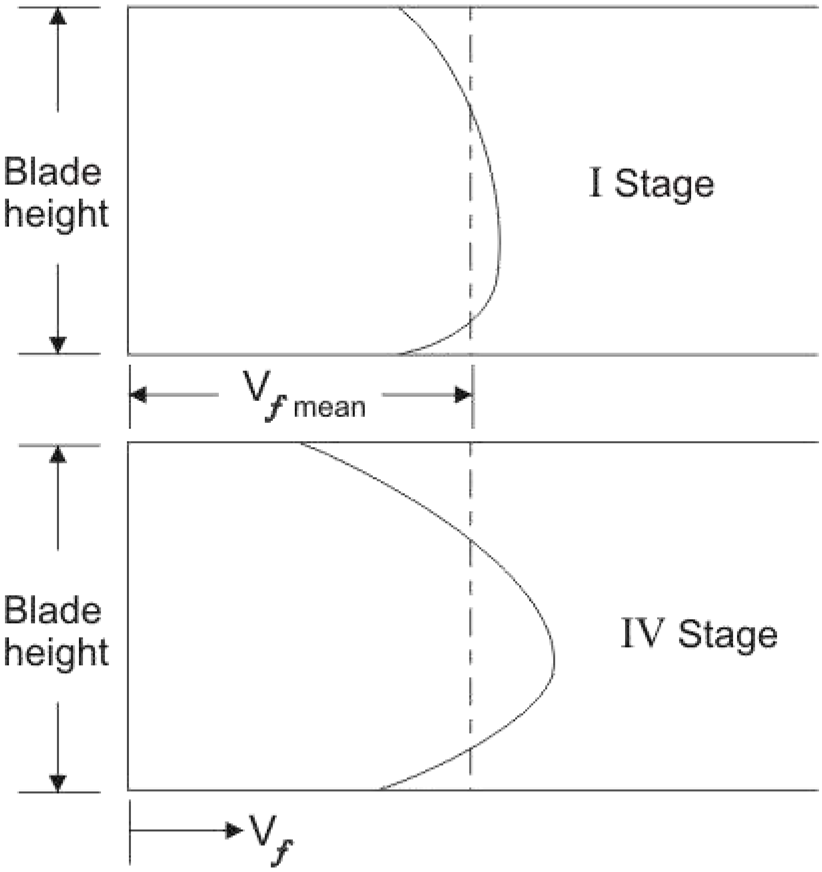
$$\Delta T_0 = \Delta T_s = \frac{UV_f}{c_p}(\tan\beta_1 - \tan\beta_2) \quad (4.15)$$

This input energy will be absorbed usefully in raising the pressure and velocity of the air. A part of it will be spent in overcoming various frictional losses. Regardless of the losses, the input will reveal itself as a rise in the stagnation temperature of the air T02 . If the absolute velocity of the air leaving the stage V2 is made equal to that at the entry. V1, the stagnation temperature rise T02 will also be the static temperature rise of the stage, T03 so that

$$\begin{aligned} w &= U[(U - V_f \tan\alpha_1) - V_f \tan\beta_2] \\ &= U(U - V_f(\tan\alpha_1 + \tan\beta_2)) \end{aligned} \quad (4.16)$$

In fact, the stage temperature rise will be less than that given in Eq. (4.16) owing to three dimensional effects in the compressor annulus. Experiments show that it is necessary to multiply the right hand side of Eq. (4.16) by a work-done factor λ which is a number less than unity. This is a measure of the ratio of actual work-absorbing capacity of the stage to its ideal value.

The radial distribution of axial velocity is not constant across the annulus but becomes increasingly peaky (Figure. 4.14) as the flow proceeds, settling down to a fixed profile at about the fourth stage. Equation (4.15) can be written with the help of Eq. (4.11) as



(4.17)

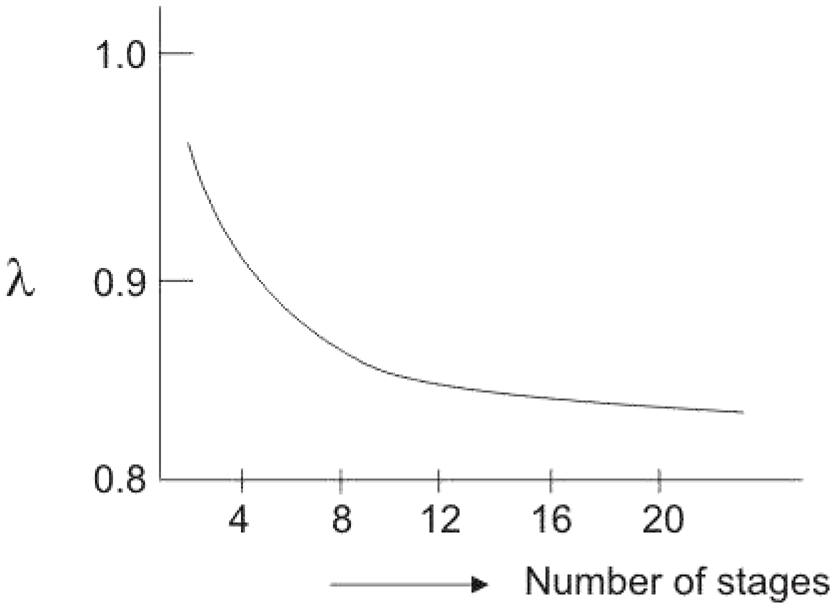


Figure 4.14 Axial velocity distribution

Since the outlet angles of the stator and the rotor blades fix the value of λ and number of stages and hence the value of V3. Any increase in number of stages will result in a decrease in λ and vice-versa. If the compressor is designed for constant radial distribution of V as shown by the dotted line in Figure (4.14), the effect of an increase in Vf in the central region of the annulus will be to reduce the work capacity of blading in that area. However this reduction is somewhat compensated by an increase in Vf in the regions of the root and tip of the blading because of the reduction of V *at* these parts of the annulus. The net result is a loss in total work capacity because of the adverse effects of blade tip clearance and boundary layers on the annulus walls. This effect becomes more pronounced as the number of stages is increased and the way in which the mean value varies with the number of stages. Care should be taken to avoid confusion of the work done factor with the idea of efficiency. If w is the expression for the specific work input (Equation. 4.13), then W is the actual amount of work which can be supplied to the stage. The application of an isentropic efficiency to the resulting temperature rise will yield the equivalent isentropic temperature rise from which the stage pressure ratio may be calculated.

DEGREE OF REACTION

A certain amount of distribution of pressure (a rise in static pressure) takes place as the air passes through the rotor as well as the stator; the rise in pressure through the stage is in general, attributed to both the blade rows. The term degree of reaction is a measure of the extent to which the rotor itself contributes to the increase in the static head of fluid. It is defined as the ratio of the static enthalpy rise in the rotor to that in the whole stage. Variation of Vf over the relevant temperature range will be negligibly small and hence this ratio of enthalpy rise will be equal to the corresponding temperature rise.

It is useful to obtain a formula for the degree of reaction in terms of the various velocities and air angles associated with the stage. This will be done for the most common case in which it is assumed that the air leaves the stage with the same velocity (absolute) with which it enters (V3).

$$\begin{aligned} w &= c_p(\Delta T_A + \Delta T_B) = c_p \Delta T_s \\ &= UV_f(\tan\beta_1 - \tan\beta_2) \\ &= UV_f(\tan\alpha_2 - \tan\alpha_1) \end{aligned} \qquad (4.18)$$

Since all the work input to the stage is transferred to air by means of the rotor, the steady flow energy equation yields,

$$w = c_p \Delta T_A + \frac{1}{2}(V_2^2 - V_1^2)$$

With the help of Eq. (10.1), it becomes

$$c_p \Delta T_A = UV_f(\tan\alpha_2 - \tan\alpha_1) - \frac{1}{2}V_f^2(\sec^2\alpha_2 - \sec^2\alpha_1)$$
$$= UV_f(\tan\alpha_2 - \tan\alpha_1) - \frac{1}{2}V_f^2(\tan^2\alpha_2 - \tan^2\alpha_1)$$

$$c_p \Delta T_A = UV_f(\tan\alpha_2 - \tan\alpha_1) - \frac{1}{2}V_f^2(\sec^2\alpha_2 - \sec^2\alpha_1)$$
$$= UV_f(\tan\alpha_2 - \tan\alpha_1) - \frac{1}{2}V_f^2(\tan^2\alpha_2 - \tan^2\alpha_1) \quad (4.19)$$

The degree of reaction

$$\Lambda = \frac{\Delta T_A}{\Delta T_A + \Delta T_B} \quad (4.20)$$

With the help of Eq. (10.2), it becomes

$$\Lambda = \frac{UV_f(\tan\alpha_2 - \tan\alpha_1) - \frac{1}{2}V_f^2(\tan^2\alpha_2 - \tan^2\alpha_1)}{UV_f(\tan\alpha_2 - \tan\alpha_1)}$$

and

By adding up Eq. (4.19) and Eq. (4.20) we get

$$\frac{2U}{V_f} = \tan\alpha_1 + \tan\beta_1 + \tan\alpha_2 + \tan\beta_2$$

$$\Lambda = \frac{V_f}{2U}(\tan\beta_1 + \tan\beta_2) \quad (4.21)$$

As the case of 50% reaction blading is important in design, it is of interest to see the result for Λ =0.5

,

$$\tan\beta_1 + \tan\beta_2 = \frac{U}{V_f}$$

and it follows from Eqs. (9.1) and (9.2) that

i.e. $$\tan\alpha_1 = \tan\beta_2, \quad (4.22a)$$

i.e. $$V_f = V_1 \cos \alpha_1 = V_3 \cos \alpha_3 \qquad (4.22b)$$

